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INVESTIGATION OF ANTENNAE BY MEANS OF MODELS

BY

J. TYKOCINSKI-TYKOCINER

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BY

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ENGINEERING EXPERIMENT STATION

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LIST OF SYMBOLS

n = Number of conductors
r = Radius of wire in cm.
d = Distance separating the conductors in cm.
h = Height above ground in cm.
b = Length of wire in cm.
m = Model coefficient.
C_o = Static capacity of original antenna top wires
C_m = Static capacity of antenna model top wires
L_o = Inductance of original antenna
L_m = Inductance of antenna model
R_o = Ohmic resistance of original antenna
R_m = Ohmic resistance of antenna model
ρ = Resistivity
μ = Permeability
I = Effective current at the voltage node of the antenna
i = Effective current at any point along the antenna
γ = Angle between two antenna top wires
λ = Wave-length in cm.
f_o = Fundamental frequency of original antenna
f_m = Fundamental frequency of antenna model
h_g = Geometric height of antenna
h = Effective height of antenna
β = Current distribution factor of antenna
C_{eff} = Effective capacity of antenna
L_{eff} = Effective inductance of antenna
S_o = Radiation resistance of original antenna
S_m = Radiation resistance of antenna model
l = Total geometric length of a circuit
C = Capacity of a standard condenser
C_s = Static capacity of antenna models
R_t = Total effective resistance of antenna
D = Distance between transmitting and receiving antenna models
α = Angle indicating setting of a condenser
δ = The smallest distance between condenser discs
a_1, a_2, a_3, a_4 = Partial areas of condenser discs

INVESTIGATION OF ANTENNAE BY MEANS OF MODELS*

I. Introduction

1. *Rôle of Antennae and the Propagation of Electromagnetic Waves.*—The most essential parts of a radio station are the transmitting and receiving antennae, and the radiation as well as the propagation of electromagnetic waves over the surface of our globe constitute the fundamental phenomena underlying the art of radio communication.

Every antenna consists of a system of wires supported on masts or towers and extending above the surface of the earth. During the process of radio transmission electromagnetic waves are detached from the antenna and forced to radiate into the surrounding space.

2. *The Development of Modern Transmitting Antennae.*—The development of the modern transmitting antennae from the Hertzian radiator[1] is represented in Fig. 1 by a sequence of diagrams.

The simplest radiating system (a) a pair of metallic rods separated by a spark gap, was extended (b) by the addition of plates or spheres, allowing larger quantities of electricity to accumulate from the charging source in order to set into play in the oscillating discharge larger amounts of radiating energy. In the hands of Marconi and other pioneers of radio telegraphy, this Hertzian radiator was set vertically (c) and assumed the largest possible dimensions. For one of the spheres the earth's globe itself was used while the dimension of the other conductor was limited only by the cost of the elevated structure and by the waste of oscillating energy occurring in the spark. The next step was therefore to exclude the primary source of oscillations from the antenna by the introduction of coupled circuits. Thus the way was cleared for further developments. Not being able to go on indefinitely increasing the height of the antenna, what was simpler than (d) to multiply the number of conductors?

*This investigation was started in December, 1922. The results were announced and the models demonstrated on December 13, 1923 in a lecture given before a joint meeting of the Urbana Section of the American Institute of Electrical Engineers, the Electrical Engineering Society, and the Physics Colloquium at the University of Illinois. The models were also demonstrated at the Electrical Show given at the University of Illinois in April, 1924.

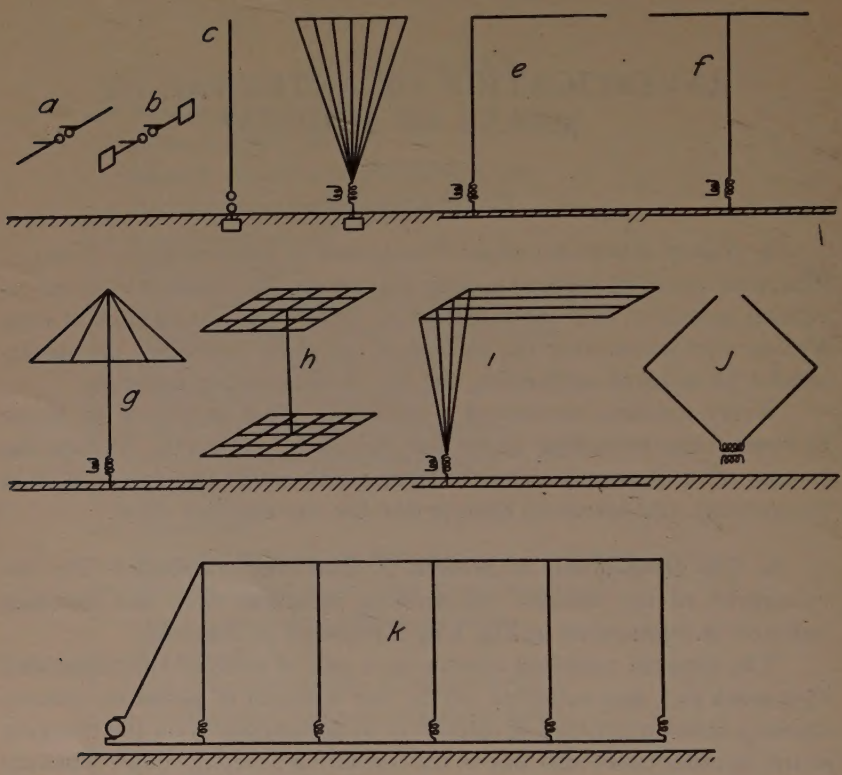

FIG. 1. DEVELOPMENT OF MODERN TRANSMITTING ANTENNAE FROM THE HERTZIAN OSCILLATOR

At this stage of development in radio communication a new demand had to be complied with besides that of long range, namely, that of selectivity. A period of extensive experiments with different forms of antenna followed resulting in radiating systems like the inverted L antenna (e), T antenna (f), umbrella antenna (g), all of them representing attempts to find empirically the optimum conditions between maximum radiating power and minimum of damping. These experiments led to an attempt by Lodge and others to return to the original Hertzian oscillator (h) by discarding the earth connection and placing large counterpoise plates, nets, or screening wires so high above the ground as to eliminate the losses due to earth currents. However, the detrimental effects due to the use at the transmitter of damped oscillations could not be eliminated by any of these radiating systems.

That is why the period immediately preceding the advent of continuous wave transmitters was marked by the development of a variety of directional antennae, [10, 11] of which the forms (i) and (j) are characteristic. With these antennae a larger degree of selectivity could be achieved. When high-frequency alternators and thermionic devices began to dominate the field of production of radio-frequency currents, the radiated wave form could be made independent of the antenna decrement and recourse was again had to multiple antennae, but in a different, more efficient form. Instead of uniting the component antennae and feeding them at the bottom (d), Alexanderson[18] introduced a multiple radiating system (k) combining a number of antennae distributed in a plane directed towards the receiver, each of the individual antennae being tuned to the same frequency and fed at the top from a common continuous wave source. Recent developments show signs of a radical change in antenna construction, with marked tendencies towards the use of small directive antennae.

To give an idea of the dimensions of the modern radiating systems and of the structures required to support them, data of the largest radio-stations are compiled in Table I.

TABLE 1
ANTENNAE DIMENSIONS OF LARGEST RADIO STATIONS

Name of Station	Country	Transmission System	Number of Masts	Height of Masts meters	Effective Height of Antenna meters	Area of Antenna System sq. m.	Capacity e.s.u.
Radio Central L. I.....	U. S.....	Alternators	6	123	85	98 300	47 700
New Brunswick, N. J...	U. S.....	Alternators	13	123	65	320 000	59 400
Tuckerton, N.J.	U. S.....	Alternators	1	305	73	29 200	39 600
Leafield (Oxfordshire)	England..	Arc.....	10	100	70		
Bordeaux.....	France...	Arc.....	8	250	170	480 000	
Lyons.....	France...	Alternators	8	200		910 000	
Nauen.....	Germany.	Alternators	6	2 x 260 4 x 120	150	156 000	30 000
Eilvesee.....	Germany.	Alternators	7	1 x 250 6 x 122		400 000	16 000 34 700
Kotwijk.....	Holland..	Alternators	6	210		450 000	30 000

3. *Independence of the Wave Propagation of the Shape of the Antenna.*—The problem of propagation of electromagnetic waves, important as it is, is entirely independent of the antenna which radiates these waves. The intensity of the oscillations in the antenna influences only the amplitude of the initial wave radiated into space, while the form of the antenna influences by its directional properties the character of the initial wave-front. For the case of a straight simple antenna this wave-front is a circle with the antenna wire in its center. For other antenna types, the wave-front deviates from the circular form, becoming an elongated closed curve for radiating systems designed to concentrate the radiation in special directions. Once a portion of energy is detached from the antenna and transmitted in the form of a wave into space, it ceases on its journey outward to be influenced by the properties of the radiating system which originated the wave. Wave propagation depends on the physical properties of the medium through which the wave travels. It is, therefore, desirable to study the properties of antennae quite apart from the propagation of waves over long distances.

4. *Difficulties Encountered in Experimental Research of Antennae and in Teaching of Principles Underlying Radio Communication. Purpose of Present Investigation.*—Considering the structures required to support radiating systems for radio communication, one can realize the obstacles encountered in any experimental investigation of the properties of different forms of antennae. An extensive study of such systems is difficult on account of the prohibitive expense involved in the erection of the antennae systems and also on account of technical difficulties connected with varying some of the elements of a system while maintaining the others constant.

Large antennae are built only for practical purposes of radio communication. Once erected such plants must be commercially operated, and under the usual circumstances not much time is left for scientific investigations.

In the domain of teaching the principles of radio communication the knowledge of the specific characteristics of radiating systems becomes more and more important. Very little, however, can be offered to the student in the way of experiment, if the usual inaccessible and invariable antenna of a college broadcasting station is used for laboratory work.

These considerations have led to an attempt to replace the full-sized antennae by models which would permit of the demonstration of all properties of radiating systems and the study of all antenna characteristics in a special laboratory.

5. *Acknowledgments.*—The author wishes to acknowledge his indebtedness to Professor E. B. Paine, Head of the Electrical Engineering Department, whose encouragement made it possible to carry out this investigation. Acknowledgment is also due to Professor A. P. Carman and Professor Morgan Brooks for reading the manuscript and for suggestions which aided in a clearer presentation of the results. In performing the measurements connected with antenna radiation the author was assisted by the able help of Mr. Lloyd P. Garner and Mr. D. Ramadanoff, both students in Electrical Engineering.

II. Principle of Similitude

6. *Scientific Foundation of the Principle of Similitude.*—Let us see first what scientific basis there is for the anticipation that the study of antennae models can disclose properties which are quantitatively and qualitatively co-related with the properties of the full-sized antennae.

A complete theory of antennae which could be applied to the calculation of radiating systems of any type does not yet exist and it is not likely that the difficulties involved will be overcome soon. Max Abraham[9] in Germany evolved a theory of a long-rod-shaped Hertzian oscillator, which represents an idealized simple antenna, and G. W. Pierce[5] in this country extended this theory to the case of a flat-top antenna. Twenty-five years ago, in his dissertation on "Electric Oscillations Around a Rod-shaped Conductor, Treated According to Maxwell's Theory," Abraham deduced mathematically about twelve fundamental properties of oscillators of this ideal type. Two of those theorems which directly concern the present problem may be mentioned.

> "The natural periods of geometrically similar oscillators are related to each other as the lengths of respective segments."
>
> "The natural periods of geometrically similar oscillators possess an equal logarithmic decrement."

The first of these two theorems states that for two geometrically similar oscillators, one of which has its geometric dimension m times smaller than the other, the wave-length emitted by the smaller will also be m times smaller than that emitted by the larger. The other

theorem states that the logarithmic decrement of both these oscillators will be exactly equal. In his theory Abraham assumed the ohmic resistance of the oscillator to be zero, so that the decrement of the oscillator would be due exclusively to radiation. It follows that geometrically similar oscillators having negligible ohmic resistance have equal radiation, no matter how large or small they are made. Such a conclusion appears strange at first, but we will find Abraham's theoretical conclusions in agreement with experiment when we consider the fact that the radiation of an oscillator increases with the frequency at the same rate at which it decreases with diminishing length.

7. *Examination of the Principle from the Engineering Point of View.*—These two theorems could be regarded as establishing a general principle of similitude for all antennae. However, it is necessary to gather sufficient evidence to prove that it applies to all oscillators, including antennae of all types.

Before the experimental investigation of models was begun, it was clear that even although a simple relation between the model and the original antenna were not discovered the study of models promised, nevertheless, to disclose sufficiently interesting material in connection with properties of radiating systems. To establish a basis for the work, known formulae having any relation with the processes going on in antennae were examined. Although rigorous proof was not obtained therefrom that it would be possible to make conclusions about all properties of a full-sized antenna from the behavior of a model, sufficient evidence was collected to encourage the investigation.

An antenna differs from any other electrical circuit only by its ability to dissipate a considerable portion of its energy in the form of electromagnetic waves which are detached from the antenna and emitted into the surrounding space. Besides this specific property, which is the radiation resistance, it possesses ohmic resistance, capacity, and inductance.

The geometric configuration of an antenna is, however, so complicated as compared with a straight wire, a coil, or a loop, for which strict formulae can be obtained that it seems hopeless to attempt to derive mathematically relations for the constants of complete antennae. But, geometrically, the most complicated antenna is a combination of these simple forms; and it should be possible to show that at least for these elements taken separately and for their mutual interaction the principle of similitude holds. By a few examples it can be shown how simple this examination can be made.

8. *Capacity of Antennae Models.*—Take, for instance, a known formula for the capacity of the upper horizontal part of a T or inverted L antenna, where

n = number of wires
r = radius of wire in cm.
d = distance separating the wires in cm.
h = height above ground in cm.
b = length of wire in cm.

The capacity of the original antenna is

$$C_o = \frac{b\,n}{2\log_e \frac{2^n h^n}{r(nd)^{n-1}} + \frac{1}{3}(n-1)\left(\frac{nd}{2h}\right)^2}$$

The capacity of the model made in scale 1 : m will be, therefore,

$$C_m = \frac{\frac{b}{m}n}{2\log_e \frac{2^n\left(\frac{h}{m}\right)^n}{\frac{r}{m}\left(n\frac{d}{m}\right)^{n-1}} + \frac{1}{3}(n-1)\left(\frac{n\frac{d}{m}}{2\frac{h}{m}}\right)^2}$$

$$\frac{C_m}{C_o} = \frac{1}{m}$$

A similar examination of a number of such formulae leads to the rule: the ratio of the static capacity of the model to that of the original system is equal to the ratio of its geometric dimensions to those of the original.

9. *Inductance and Mutual Inductance of Antennae Models.*—For the inductance and for the mutual inductance of some systems, the same rule can be deduced, simply because in the formulae involved the dimensions of the system appear in terms of ratios in which the model coefficient m is eliminated, leaving only the factor $\frac{1}{m}$.

The same rule applies also to loop antennae. The loops used for transmitting or for reception are but large coils of special form. If we

examine the universal formulae given by Brooks and Turner[14] for the inductance of coils, we will find the same rule of models applicable to coils of any shape.

When the high frequency resistance of conductors and the influence of skin effect upon the inductance and resistance of antennae are taken into account, correction factors will have to be introduced for the model coefficient m. Under certain conditions, however, these corrections can be made of negligible magnitude.

Take for instance the Heaviside-Rayleigh formula for the inductance of a straight round wire for high frequency having a length b, radius r, resistivity ρ, and permeability μ. The decrease of inductance due to the action of the magnetic field inside the conductor is:

$$\Delta L_o = -\frac{\mu b}{2}\left(1 - \sqrt{\frac{\mu 2\pi f r^2}{\rho}}\right)$$

For a model with coefficient m this would become

$$\Delta L_m = -\frac{\mu b}{2m}\left(1 - \sqrt{\frac{\mu 2\pi m f r^2}{\rho m^2}}\right)$$

The decrease of inductance with frequency is not a linear function of m. Only for a model in which

$$r_m = \frac{r_o}{\sqrt{m}}$$

where r_m is radius of wire for model, and r_o radius of wire for antenna, is ΔL_m proportional to m

so that

$$\Delta L_m = \frac{\Delta L_o}{m}$$

10. *Ohmic Resistance of Antennae*[19] *Models.*—As to the ohmic resistance of the antenna it can be shown that in order to obtain the same value for the model as for the original, it would be necessary to decrease the resistivity of the conductors in the same proportion as the geometrical dimensions. This can be achieved for the conducting layers below the antenna (for the earth), but for the wires a much more conductive material than copper is not available.

According to a formula, also given by Heaviside and Rayleigh, the resistance of straight round wire is:

$$R_o = \sqrt{\frac{\mu f b^2 \rho}{r^2}} = \frac{b}{r}\sqrt{\mu f \rho}$$

For the model this would become

$$R_m = \frac{b\,m}{r\,m}\sqrt{\mu\, m f \rho}$$

whence

$$R_m = R_o \sqrt{m}$$

If, however, for the radius ratio of the wires a coefficient $\sqrt{m}$ be chosen we obtain for the resistance of the model

$$R_m = \frac{R_o\sqrt{m}}{\sqrt{m}} = R_o$$

that is, a resistance exactly equal to that of the original system. So in those cases where the determination of the ohmic resistance of antennae is of more importance than the determination of the other constants a radius ratio equal to not m but the square root of m should be employed for those wires which carry the largest currents.

11. *Radiation of Antennae.*—Turning our attention to the radiation of antennae, which is the most important characteristic from the point of view of radio engineering, the question arises: Is there any chance of obtaining with models of the usual antennae types the same radiation coefficients as those of the original antennae? Can the rule for models deduced from Abraham's theoretical investigation of linear Hertzian oscillators be extended to antennae used in radio telegraphy? There exist indications both of a theoretical and of an empirical character that under certain restrictions this is the case.

The power radiated from an antenna is

$$P = I^2 S_o$$

The factor S_o, called the radiation resistance of the antenna, is, unlike its ohmic resistance, independent of the conductivity of the material of

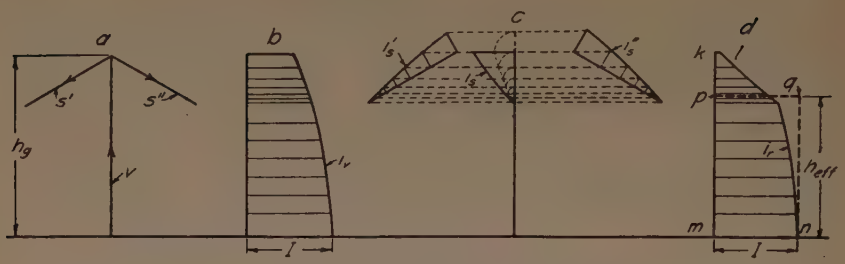

FIG. 2. CURRENT DISTRIBUTION AND EFFECTIVE HEIGHT OF AN ANTENNA

which the antenna is made. Its relation to the emitted wave-length and to the geometrical configuration of the antenna is given by the expression

$$S_o = 160\,\pi^2\,\frac{1}{\lambda^2}\left[\frac{1}{I}\int idh_g\right]^2 = 160\,\pi^2\,\frac{h_{eff}^2}{\lambda^2}$$

where λ is the wave-length, I is the current at the foot of the antenna, idh_g indicates the distribution of the current along the antenna, h_g is the geometrical height of the antenna, which is assumed to be considerably smaller than the wave-length, and

$$h_{eff} = \frac{1}{I}\int idh_g$$

is the effective height of the antenna.

Figure 2 serves to show graphically the relation between the geometric height of the antenna and its effective height. In the figure (a) is an antenna consisting of a vertical conductor v and two suspended conductors s' and s'', (b) shows the distribution of current along the vertical conductor, (c) gives, by means of a geometric construction, the sum of the vertical projections of current elements for the two suspended conductors, and (d) shows the resulting current distribution for the entire antenna. The area $k\,l\,m\,n$ divided by the current I measured at the foot of the antenna gives the effective height

$$h_{eff} = \beta h_g$$

This distribution factor β can assume all values between 1 and 0 and depends solely on the geometry of the antenna. So for a vertical antenna with an extended system of wires at the top having a very large capacity

β approaches unity, which means that the current in this case is nearly equal all over the length of the antenna. For a simple vertical antenna having its inductance and capacity per unit length constant over the entire length, the current distribution curve is sinusoidal and

$$\beta = \frac{2}{\pi}$$

For the same type of antenna, but with a loading coil of large inductance inserted at the point of maximum current, the current distribution curve is a straight line and

$$\beta = \frac{1}{2}$$

12. *Radiation of Antennae Models.*—In all these cases the relative position and distribution of capacity and inductance along the antenna determines the value of the factor β and the absolute values of the geometric dimensions of the antenna have no influence whatever upon the current distribution. It may be concluded, therefore, that for any given type of antenna the current distribution does not vary if all of the geometric dimensions of the antenna are increased or diminished in the same proportion.

Considering that, if C_{eff} is the effective capacity and L_{eff} the effective inductance of the antenna,

$$\lambda^2 = (2\pi)^2 C_{eff} L_{eff}$$

and

$$h^2_{eff} = \beta^2 h_g^2$$

the expression for the radiation resistance of an antenna assumes the form

$$S_o = \frac{160\,\pi^2\,\beta^2\,h_g^2}{(2\pi)^2\,C_{eff}L_{eff}} = 40\,\beta^2\,\frac{h_g^2}{C_{eff}L_{eff}}$$

The radiation resistance of a precise model of the original antenna having all dimensions diminished m times is accordingly

$$S_m = \frac{40\,\beta^2\left(\frac{h_g}{m}\right)^2}{\frac{C_{eff}}{m}\,\frac{L_{eff}}{m}}$$

so that

$$S_o = S_m$$

It may be expected, therefore, that for all types of antennae the rule holds that the radiation resistance of a model antenna is approximately equal to that of the original antenna.

13. *Fundamental Frequency of Antennae Models.*—The introduction of the terms effective capacity C_{eff} and effective inductance L_{eff} of antennae makes it possible to apply Thompson's formula for the calculation of oscillation frequencies of antenna circuits. For a rationally designed antenna the radiation resistance S_o constitutes the predominant part of its total resistance. Neglecting the ohmic resistance the frequency of the original antenna is

$$f_o = \frac{\sqrt{\frac{1}{L_{eff} C_{eff}} - \frac{S_o^2}{4 L^2_{eff}}}}{2\pi}$$

and the frequency of the antenna model

$$f_m = \frac{\sqrt{\frac{m}{L_{eff}} \frac{m}{C_{eff}} - \frac{m^2}{4 L^2_{eff}} S_o^2}}{2\pi} = m f_o$$

This indicates that the oscillation frequency of the model increases with the model coefficient m. The smaller the model is made the higher becomes its fundamental frequency as compared with that of the original antenna.

III. Production and Measurements of Short Electromagnetic Waves

14. *Antenna Models and the Application of Very High Frequencies for Their Excitation.*—The use of a standard wave-length of 500 or 350 meters for the excitation of antennae models would necessitate the introduction of very large loading coils in the model. This would reduce the radiation to a negligible magnitude, and by no means could the real relations be deduced from such an experiment.

The use of much shorter waves becomes imperative and this brings the story back to 1887, the time of Hertz's classical experiments, when wave-lengths of a few meters were used in most of the researches. The progress made by science since 1887 gives the present day investigators

great advantages over those of that period. They need not be limited by the use of sparks and damped oscillations for the production of short waves; the hot cathode vacuum tube also affords a most simple and convenient means of producing Hertzian waves.

The first experiments in the production of undamped Hertzian waves by means of hot cathode tubes were made by White[26] of the General Electric Company in 1916. Since then a number of schemes have been proposed in this country[29] and abroad.[27, 28, 30] However, a number of modifications were necessary in order to obtain reliable transmitters for the excitation of antennae models.

15. *Principle of the Transmitter for the Production of Frequencies of from 50 to 100 Millions per Second.*—It is very well known that in order to make a three-electrode tube oscillate, it is necessary to choose the conditions of the circuits so that at any moment the a-c. potentials of the plate and grid in respect to the filament will be in opposite phase. This condition enables the initial impulse in the oscillating circuit to be amplified and supported by consecutive impulses. In ordinary long-wave oscillating arrangements this is done conveniently by proper connection of the so-called feed-back circuits coupled with the oscillating circuit for the purpose of diverting a fraction of the oscillating energy to the grid circuit. However, in applying the same rule to the production of ultra radio frequencies, peculiar properties of circuits become prominent and must be taken into account. Consider a closed circuit (Fig. 3a), with its capacity and inductance partly lumped in condenser c and coils L, and partly distributed along the leads CL. If the frequency is so low that the geometrical length of the circuit is small compared with the wave-length produced, the current will be the same in every section of the circuit. The usual radio frequency circuits have a length l about 50 to 500 times smaller than their wave-length λ. The current distribution along the circuit is therefore uniform and an ammeter a inserted at any point of such a quasi-stationary oscillating circuit will show the same current everywhere. But when with increasing frequency the ratio l/λ becomes larger, the uniformity of the current distribution gradually disappears and, for a frequency sufficiently high, distinct standing waves may build up along the circuit. These waves are illustrated graphically in Fig. 3b by a dotted curved line drawn along an identical circuit but opened in C and extended for the purpose of clearer representation. The current I, measured along such a circuit from point 1 to point 9, shows different values varying from zero at the nodes to maximum values at the antinodes.

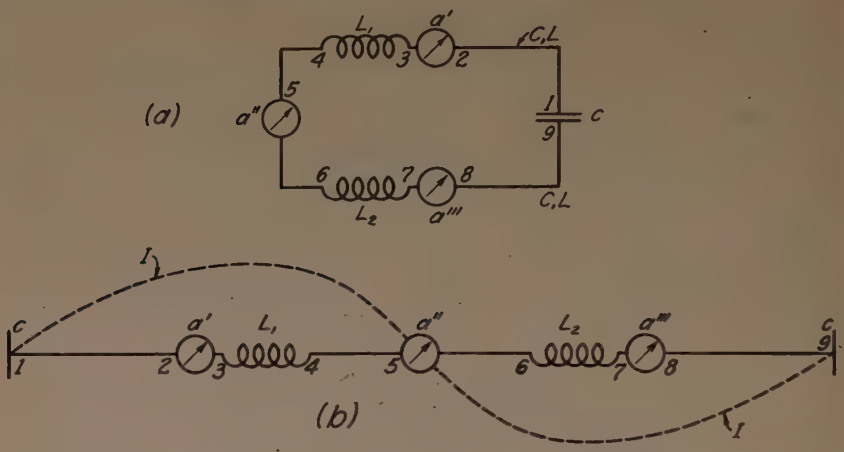

FIG. 3. CIRCUIT WITH NON-QUASI-STATIONARY CURRENT DISTRIBUTION

To avoid complications in the transmitter caused by the use of a combination of circuits in which the current, due to the very high frequency cannot be made quasi-stationary, the usual feed-back circuits were discarded altogether.

Figure 4 shows diagrammatically the principle of the arrangement of the transmitter. An oscillating loop is divided symmetrically into two parts, of which the part L_1, which is used for coupling with a Hertzian oscillator h, with an antenna, or with any other circuit to be investigated, is visible outside the transmitter, and the other part L_2 is concealed behind a bakelite board. The two parts of the loop are joined with each other by means of two condensers. One of the condensers C_o is variable, having a capacity range of from 5 to 15 e. s. u. It consists of two semicircular plates 7.5 cm. in diameter separated by 1½ mm. air space. The other condenser C_v is formed by the plate and the grid of the vacuum tube V and has a capacity of about 6 e. s. u. Choking coils a, b, and c are inserted in the leads connecting the plate circuit and the filament circuit with their batteries for the purpose of suppressing the propagation of oscillations from the loop along these leads.

A comparatively large condenser C_2 (250 e. s. u.) is inserted in the middle position of the loop to prevent a direct connection of the plate with the filament. The thermo-ammeter A_t is inserted in a convenient place to indicate the maximum effective current in the loop. The zero

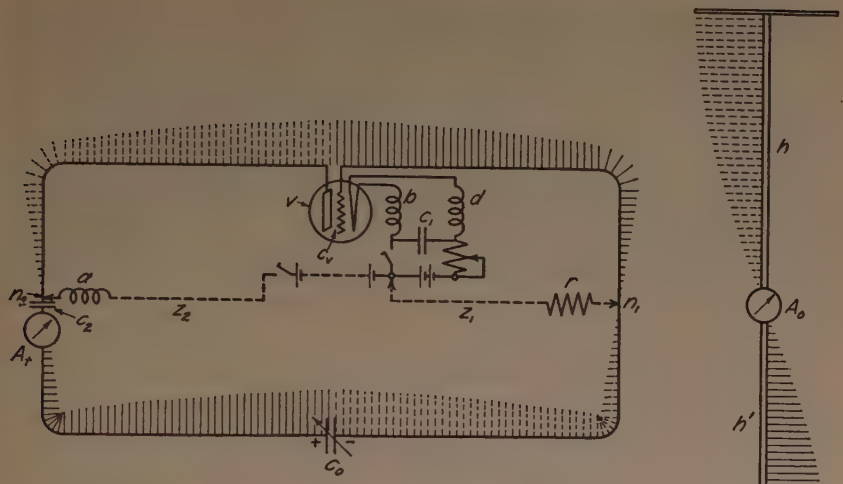

FIG. 4. DIAGRAM OF THE SHORT WAVE TRANSMITTER

potential leads z_1, z_2 connecting two symmetrical nodal points n_1, n_2 on the loop contain the grid leak r of about 10 000 ohms resistance on the grid side and choking coil a on the plate side. Standard 1-watt and 5-watt vacuum tubes can be used for obtaining ultra radio-frequency oscillations, but not all of them work equally well. Tubes that are too soft do not oscillate at all or are very inefficient.

By means of an insulated exploring wire the nodal points n_1 and n_2 can easily be found and the points where the zero potential leads z_1 and z_2 must be attached are thus determined. The amplitude of oscillation is rapidly decreased by shifting these leads in the direction toward the condenser C_o or C_v. Proper selection of the points for connecting the leads z_1 and z_2 and proper adjustment of the condenser C_o are therefore essential for the efficient production of oscillations.

The distribution of potential along the loop is indicated in Fig 4 by the fine lines drawn perpendicularly to the heavy line representing the wire of the loop. The length of the lines represents the relative potential difference with respect to the nodal points. Full lines drawn to the right of the vertical parts of the oscillator represent positive

FIG. 5. SHORT WAVE TRANSMITTER COUPLED WITH HERTZIAN OSCILLATOR

values, dotted lines drawn to the left negative values of the potential at the same moment. It is evident from this diagram that the potential at the plate and grid are opposite in phase by nearly 180 deg.

Under these operating conditions frequencies of from 50 to 100 millions per second are easily obtained, corresponding to a wave-length of 6 to 3 m. For longer wave-lengths the loop is replaced by rectangular coils.

16. *Description of the Transmitter.* —The complete transmitter is shown in Fig. 5. The coupling loop is visible at the left. It can be set in any angular position with respect to an antenna or to any secondary circuit to be excited. To the right of the loop a number of controlling instruments are mounted on a bakelite board.

First row of instruments:

- Variable condenser
- Radio frequency thermo-ammeter
- Plate current ammeter
- Filament current ammeter
- Switch of plate circuit
- Switch of filament circuit

Second row of instruments:

- Grid-leak resistances
- Choking coils
- Filament circuit condenser
- Positive terminal for plate voltage
- Thermionic oscillating tube
- Condenser
- Filament rheostat
- Filament rheostat for fine adjustment
- Filament battery terminals

The bakelite board with these instruments mounted on it is fixed to two supports. One of these carries wooden bearings for a shaft, the rear end of which has a knob attached to it, while the front end has a thermo-ammeter mounted on a board. The terminals of this ammeter are supplied with two brass tubes into which a set of rods, tubes, and spheres of different dimensions are made to fit, forming Hertzian oscillators. By means of the knob at the back of the transmitter, the Hertzian oscillators can be rotated from the horizontal into the vertical position.

The fundamental period of the Hertzian oscillators is adjustable by shifting the tubes telescopically. For experiments with antennae or with closed circuits, the complete Hertzian oscillator is easily removed from the transmitter by withdrawing the shaft from its bearings.

17. *Distribution of Current and Peculiarities of the Transmitter Circuit.*—The following experiments can be easily performed to demonstrate the behavior of the short wave transmitter. The most striking phenomenon is the distribution of potential, which is by no means quasi-stationary. A metal rod a few inches in length, and supplied with an insulated handle can be used to explore the circuit. The rod concentrates on its surface lines of electric force from the surrounding field, and, like a condenser, is being charged whenever approached to or brought in contact with any part of the transmitter loop. When a point of zero potential is touched, the exploring rod takes up no charge, and no influence whatever can be noticed in the operation of the transmitter. If, however, other points of the loop are touched with the rod four effects are produced: A certain quantity of energy is translated from the circuit and concentrated around the rod, the effective capacity, and with it the natural period of the circuit, is increased. A redistribution of the current and of the potential along the circuit takes place, the potential at the grid is also affected, and consequently the amplitude of oscillations must change. All these effects cause, therefore, a variation of the current indicated by the thermo-ammeter inserted in the circuit. When the exploring rod is shifted along the loop, if for certain regions maximum variation of current is noticed, it may be concluded that the rod is touching a point of highest potential (current node); if, on the other hand, in another region of the loop the current variations are seen to decrease rapidly, it denotes that the rod is approaching the point of lowest potential (current antinode). By observing the rate at which the current varies while the rod is continuously moved along the loop, a relative measure of current and potential distribution from point to point is obtained.

The shifting of the neutral leads z_1, z_2 along the loop produces similar effects. Instead of observing variation of the oscillating current, the observer can determine from the indications of the plate current ammeter and of the plate voltmeter whether the points touched by the exploring rod or by the neutral leads are in the region of a node. The efficiency of the transmitter is maximum when the neutral leads are connected to the loop exactly at the nodal points of potential.

The variation of the capacity of the condenser C_o gives a means of changing the frequency of the oscillating circuit. But this variation of capacity changes also the distribution of current and potential so that variations of amplitude follow for exactly the same reasons as already mentioned. If the effective capacity of the condenser C_o is much smaller or much larger than the capacity of the oscillating tubes, the oscillations cannot be sustained in either case.

A remarkable effect of the filament heating current has been observed. The ammeter indications in the oscillating circuit increase rapidly with the increase of filament temperature, but at a certain critical filament current they suddenly fall off, followed by a drop in the plate current and a rise of plate voltage. By using the exploring rod it can be shown that a sudden radical redistribution of current along the transmitter loop is brought about, with the number of nodes increased whenever the filament heating current exceeds a certain value. This effect is especially marked in the case when the lead z_1 and the grid resistance r are removed.

The frequency of the oscillations produced by the method described is constant and depends only on the effective capacity and the inductance of the loop and the apparatus connected with it. Because of this constancy, and of the possibility of measuring the emitted wave-length with considerable accuracy, this method of producing oscillations of constant amplitude and frequency can be utilized for a great many investigations in physics and electrical engineering.

18. *Absolute Measurement of Wave-Length.*—The principles of wave measurement are exactly the same as used in radio telegraphy and Hertzian wave investigations, but the methods have to be changed somewhat in view of the special applications.

In the practice of radio telegraphy a variable condenser of known capacity and a coil of known inductance combined with a hot wire ammeter or thermo-galvanometer are all of the apparatus necessary for the determination of frequency. The capacity of the coil is in most

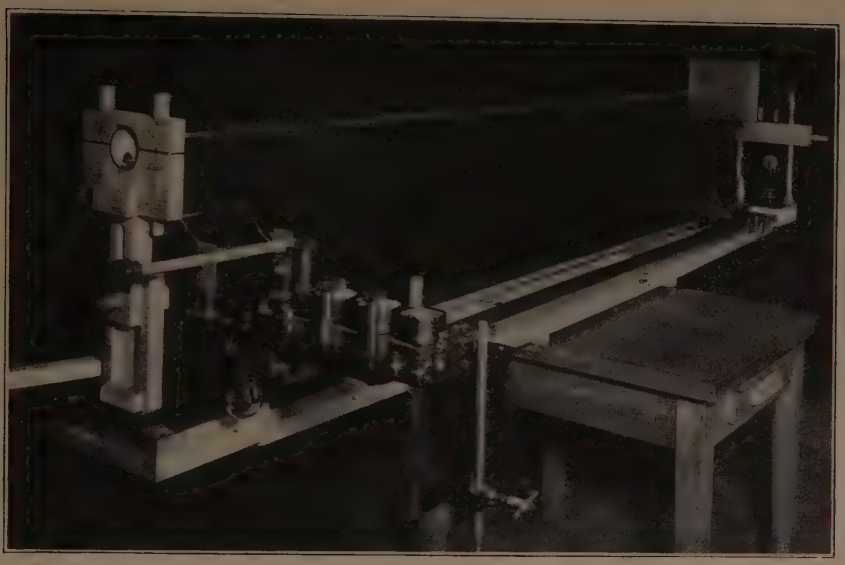

FIG. 6. THE WAVEMETER BRIDGE FOR ABSOLUTE MEASUREMENT OF WAVE-LENGTH

cases negligible as compared with the capacity of the condenser, its resistance is also neglible, and the circuit is quasi-stationary, so that Thomson's formulae can be applied without correction.

In dealing with circuits whose natural frequency is very high, about 60 millions per second, the current, as has been shown, is not the same in all sections. The distribution of current along the circuit is usually not known, and only for ideal cases can the natural frequency be obtained by calculation. Nevertheless, we can calibrate such circuits by means of stationary waves formed along two parallel wires. Such a system of wires, originally introduced by Lecher[25] has its capacity and inductance uniformly distributed along the wires, its current distribution is sinusoidal, and the system permits of an absolute measurement of the wave-length.

The bridge shown in Fig. 6 consists of two parallel wires 10 mils (0.025 cm.)in diameter forming a loop, closed on one side with the thermocouple of a thermo-galvanometer, and on the other side with a plate set normal to the plane of the loop. When high frequency electromotive forces are induced by the transmitter at one end of the narrow loop, a succession of waves is caused to travel along the wire of the loop. These

waves reverse their direction for every alternation of the electromotive force, and each wave on its way back to the origin interferes with those going towards the plate. If the distance between the origin and the plate is exactly equal to half a wave-length or to a multiple thereof, the interference of two waves travelling in opposite directions results in a standing wave with current nodes in the middle of the system and with potential nodes at the origin and also at the end plate. By moving the plate along the wires and observing the deflection of a galvanometer the precise position of nodal points can easily be determined. The distance between the origin and the first node, or between two consecutive nodes gives the half wave-length in air of a wave radiated from the transmitting loop.

19. *Description of the Calibrating Wavemeter Bridge.*—The bridge (Figs. 6 and 7) consists of a wooden frame with two guides running along its entire length and with a pulley mounted at each end. A platform is moved along the guides by a cord running over the pulleys. By means of a screw the platform can be attached to the cord or made loose enough to be shifted directly by hand. The platform carries a horizontal plate with a sliding carriage forming the base of a bakelite support for a vertical screen made of sheet brass. By means of a micrometer screw, the position of the carriage together with the supported screen plate can be accurately adjusted. The plate has a horizontal row of fine holes to carry two fine copper wires and to keep them parallel at definite distances from each other. These parallel wires connected at one end by the thermo-galvanometer, after passing at the other end the holes in the screen, are kept under tension by a weight attached to two cords passing over rollers and joined with the wires. By moving the plate along the bridge, the parallel wires of the system become conductively connected with each other at definite distances from their origin. The magnitude of this distance is measured with a meter scale fastened to the bridge, and with a millimeter vernier fixed to the carriage. At the origin of the parallel wire system a clamp attached to a supporting rod carries the short wave transmitter with its coupling loop directed towards the wires. The indicating instrument consists of a Weston thermo-galvanometer in which a group of thermo-couples is connected with a pointer galvanometer. This instrument is mounted in a wooden holder adjustable in a sliding vertical frame so that the height of the instrument and of the two parallel wires above the transmitting loop can be adjusted, and the coupling with the transmitter thus varied. In

order to increase the total length of the parallel wire system, the support for the indicating instrument can be removed from the frame of the bridge and attached to a table at any distance without disconnecting the wires. This is done when measuring wave-lengths greater than the double length of the bridge (2.5 m.).

As an indication of the accuracy of wave-length measurements by this apparatus it may be stated that, for a frequency of 60 million cycles per second ($\lambda = 5$ m.), when the plate is shifted 2.5 cm. from the point of resonance, corresponding to a discrepancy of one per cent in wave-length, the deflection of the thermo-galvanometer is reduced 57 per cent of its resonance value. A discrepancy of one-tenth of one per cent in wave-length is still noticeable.

The distribution of the potential and current along the parallel wire system can be studied by the same method as used for the transmitter loop. However, the system is so sensitive to the slightest displacement of the wires in relation to each other that the method of touching the wires with an exploring handle cannot be used. Instead, aluminum riders with holes in the center are shifted symmetrically along the wires. With the aluminum disc (1 cm. radius) placed at a distance from the origin corresponding to one-fourth wave-length, the deflection of the instrument is reduced 75 per cent, with the disc at a distance of one-eighth wave-length, 44 per cent, and with the disc at a distance of one-sixteenth wave-length, 18 per cent.

20. *Apparatus for Demonstrating Mechanical Analogy of Wave Propagation over Wires.*—In view of the increasing importance of electromagnetic waves along wires in connection with carrier wave telephony, short wave radio communication, and electric power transmission, more attention will have to be given in the future to the demonstrating of the properties of these waves and of the laws of their propagation. As an introduction to the study of electrical phenomena, mechanical analogies have proved helpful. The wavemeter bridge has therefore been adapted to demonstrate and to investigate mechanical vibrations of wires which are in close analogy with electromagnetic oscillations so as to visualize the relations existing between certain physical quantities involved in the process of sustaining and propagating vibrations.

The wavemeter bridge as shown in Fig. 7 is supplied for this purpose with a mechanical vibrator in place of the electric oscillator. This vibrator consists of a polarized electromagnet, mounted on an adjust-

FIG. 7. THE WAVEMETER BRIDGE ADAPTED FOR THE DEMONSTRATION OF ANALOGY BETWEEN THE TRANSMISSION OF ELECTRICAL OSCILLATIONS AND MECHANICAL VIBRATIONS OVER WIRE SYSTEMS

able support. The armature has an extension in form of a rod designed for mechanical coupling with the wires to be set in vibration. In order to regulate the amplitude of motion the armature is damped by means of rubber strips or a spring applied through a lever. A pair of leads connects the electromagnet with a 110-volt alternating current source of constant frequency (60 cycles per sec.). The armature, excited by the alternating current, is set into vibration, and due to its angular (rocking) motion imparts to the stretched wire impulses which, after proper adjustment of the wire length and tension, produce standing waves of a circular type. To visualize the phase relations and the nodes, markers of white thread or white beads are distributed equidistantly along the wire. The antinodes appear to the eye as white circles of maximum diameter, the nodes are visible as brilliant points, while the intermediate marks are seen as circles of smaller diameter.

These standing waves can be photographed. Fig. 7 shows a reproduction of an experiment in which a copper wire, 0.025 cm. in diameter, under a tension of 340 gr., was set in vibration, forming 3 distinct half waves each 72 cm. in length.

In this analogy of mechanical vibrations with electric oscillations, the mass of wire per unit of length corresponds to the inductance of the wire per unit of length, and the tension (restoring force) of the wire corresponds to the distributed capacity per unit length of the wire. If, therefore, the tension is increased the effect is similar to that produced by increasing the distributed capacity of the wire and consequently when the frequency remains constant the wave-length must become

longer. In fact, with a tension of 150 grams the wire of Fig. 7 vibrates in 4 half waves; with 125 grams tension it vibrates in 5 half waves; while with 85 grams tension, 6 half waves appear over the same total length of the wire. Even as many as 15 half waves could be produced on the same wire. By varying the wire length by means of the fine adjustment screw, the phenomena of mechanical resonance can be demonstrated and studied.

In exactly the same manner two wires are set in vibration each in opposite phase forming standing waves approaching a complete mechanical analogy with a Lecher parallel wire system.

21. *Condenser for Short Wave Measurements.*—Returning to electrical wave measurements, it was found that a wavemeter bridge of the kind described is not convenient for general use in connection with experiments, where measuring instruments of small size are desirable. It is therefore used only for the calibration of compactly constructed wavemeters.

In designing wavemeters for very short waves, one meets with difficulties which are not encountered in dealing with long waves. The first difficulty is the influence of the observer and conductors near the condenser. The condenser to be used for short wave measurements must naturally be small. In the case of semicircular disc condensers, not more than two discs can be used. The stray field of such a condenser varies with the relative position of the plates. The stray field intensity is largest when the position of the plates corresponds to minimum capacity, it is smallest for the position of maximum capacity. This stray field, varying with the relative position of the plates, and also with the position of the coil, causes errors for which no simple correction can be made. The second difficulty is the necessity of very fine adjustment. For the wave-lengths usually employed in radio transmission the variable condenser carries an additional pair of plates serving as a vernier, or the entire set of movable plates may be adjusted by a worm gearing. The vernier condenser method cannot be applied in the present case because the addition of an extra plate would unduly increase the capacity. The worm gearing requires a metallic support and a number of metallic parts, causing increase of the stray field and also considerable increase of the condenser capacity.

The condenser which finally was evolved from these considerations consists of a pair of circular discs of which one can be rotated. Both inner surfaces of the discs facing each other are divided into two equal

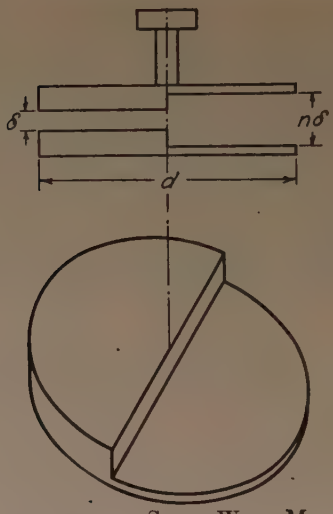

FIG. 8. CONDENSER FOR SHORT WAVE MEASUREMENTS

semicircular parallel parts, each of them on a somewhat different level as shown in Fig. 8. The step-like discs are mounted in a cylindrical bakelite box as shown in Fig. 9. Of these condenser members the stationary one *a* is fixed to the bottom of the box, while the rotating one *b* is set on a circular brass plate of somewhat larger diameter. The latter fits into the opening of the box and is pressed by means of a star-like spring *c* and by the bakelite cover *d* against a bronze ring fixed in the box above the stationary condenser member. The calculation of the capacity of this kind of condenser and a description of its properties is given in Appendix A.

22. *The Wavemeter and Methods of Wave-length Determination.*— The application of a condenser designed on this principle to a wavemeter is shown in Fig. 9. The inductance consists of one turn of brass strip bent into a narrow rectangular loop. The loop can be rotated around the condenser and brought into every position convenient for performing the measurements, without the danger of changing the constants of the wavemeter circuit. This is possible because of the symmetrical shape of the condenser.

A thermo-couple combined with a galvanometer connected parallel to a portion of the wavemeter loop causes deflections increasing to a maximum, when the circuit is approaching resonance. The constants of the wavemeter vary, however, when the position of the thermo-couple is shifted along the loop. It was found convenient, therefore,

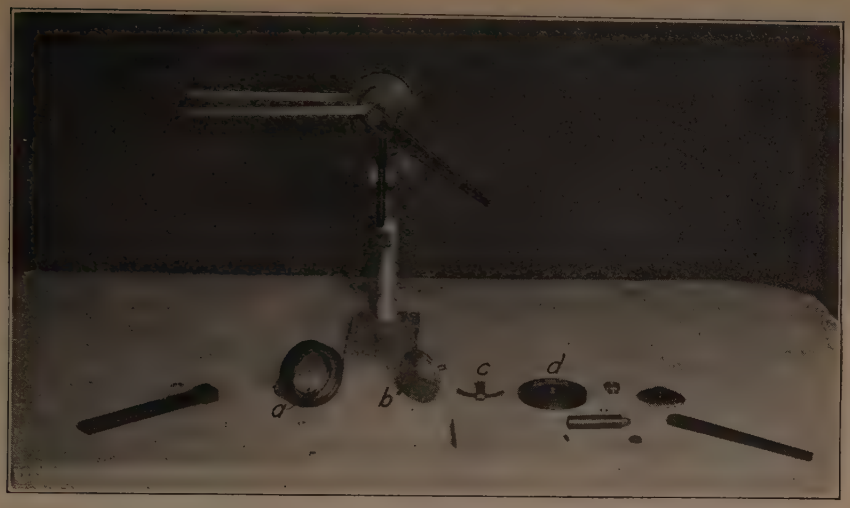

FIG. 9. WAVEMETER WITH SPECIAL CONDENSER FOR SHORT WAVE MEASUREMENTS

not to use any indicator connected directly with the wavemeter. The output of the transmitter can be adjusted so that any absorption by a neighboring circuit is easily observed by the decrease of current in the transmitting circuit. The loop of the wavemeter is directed towards the transmitter and the calibrated condenser adjusted until the ammeter in the transmitter circuit indicates minimum of deflection. This takes place when the wavemeter circuit is in resonance, absorbing maximum energy from the transmitter.

Another method of determining the resonance condenser setting of the wavemeter without the use of any indicating instrument connected with the wave measuring circuit is to observe the plate voltage or plate current feeding the oscillating tube of the transmitter. The absorption of energy by induction in the wavemeter circuit is equivalent to a resistance inserted in the transmitter oscillating circuit. The direct current transmitter input varies, therefore, during the process of tuning. Sudden changes appearing at the moment of passing the point of resonance can be observed by the indications of the plate voltmeter or ammeter.

IV. APPLICATIONS OF ANTENNAE MODELS

23. *Models for Teaching Purposes.*—Antennae models may be used either for teaching or for research purposes, especially for predicting those constants, the knowledge of which is necessary for proper designing of radiating systems adopted for radio communication.

For demonstrating the chief properties of antennae it is not required to have precise models. Simpler models without supporting masts but with rigid conductors able to carry their own weight may be used for this purpose. It is, however, useful to start the experiments with the prototype of all antennae, a Hertzian oscillator of the form used by the discoverer in his classical investigations, but omitting the cumbersome spark gap method of excitation.

24. *The Hertzian Oscillator.*—A Hertzian oscillator of this kind, consisting of a pair of telescopic tubes with spheres attached to them, can be made to radiate electromagnetic waves, if properly coupled with a short wave transmitter as shown in Fig. 5. It is necessary that the length of the oscillator tubes connecting the centers of the spheres be adjusted in resonance and placed parallel to the plane of the transmitting loop, because only in this position, when the direction of the magnetic field is perpendicular to this axis, can electromotive forces be induced along the oscillator. The currents induced in the oscillator when rotated around its bearing will vary from a maximum value when it is in the horizontal position to zero when it is placed vertically. This variation is indicated by the thermo-ammeter inserted in the center of the oscillator. In order to excite the oscillator when placed in a vertical position the loop of the transmitter must be rotated and brought also into the vertical direction.

The distribution of current and voltage along a Hertzian oscillator is the result of two waves, one traveling from the center towards the spheres, and the other reflected from the ends of the oscillator and rushing back to the center. The value of the current is zero at the oscillator ends and achieves a maximum value in the center. That this is the case can be shown by shifting the ammeter along the oscillator so that the same length of tube is always shunting the instrument. Contrary to the distribution of current, the value of the voltage is maximum at the ends and zero in the center. The distribution of voltage along the oscillator can be investigated by a method similar to that indicated for the transmitter and wavemeter bridge; or another oscillator in the form of a rod held at the center can be coupled to that under investigation and the influence of its position upon the indication of the ammeter inserted in the current antinode studied. If the rod is placed vertically to the oscillator and with one end in contact with it, the magnetic field of the oscillator cannot produce electric forces in the rod. Only by charging and discharging can the rod be excited. The charge which the rod takes

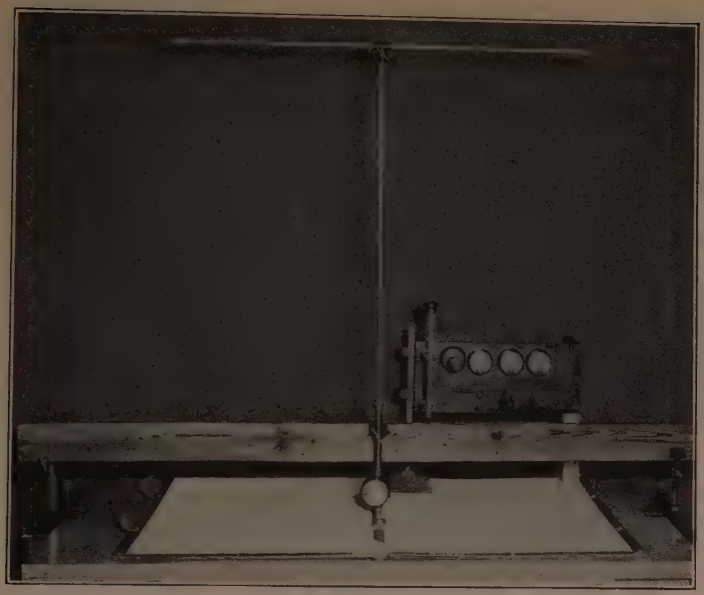

FIG. 10. MODEL OF A T ANTENNA FOR TEACHING PURPOSES

up will be largest and the greatest amount of energy will be drawn from the oscillator at points having the highest potential. By shifting the rod along the oscillator a decrease of the oscillator current is observed. With the rod at the center of the oscillator no appreciable change of current amplitude is observed because this point corresponds to a potential node. However, when the sphere of the oscillator is approached the decrease of current becomes appreciable and reaches a maximum at the end point where an antinode of potential is found. For a Hertzian oscillator made up of a thin rod of uniform thickness so that the capacity and the inductance per unit of length is constant all over the oscillator, the distribution of current and potential along the oscillator is sinusoidal. If, however, a coil of comparatively large inductance is introduced in the center, the current and potential distribution becomes linear, i.e., the rate of current decrease is then constant in every section of the rod and the potential gradient along the rod becomes also constant. On the other hand, it can be shown that large spheres attached at the ends of the oscillator, increasing considerably the capacity of the system, cause the current to approach constant values in every section of the rod.

FIG. 11. MODEL OF AN INVERTED L ANTENNA FOR TEACHING PURPOSES

By means of a Hertzian resonator combined with a sensitive thermocouple connected to a galvanometer the classical experiments of Hertz to show the existence of waves in the space surrounding the oscillator can be reproduced.

25. *The* T *Antenna and Its Transformation into Various Types.*— To demonstrate the origin and the chief properties of antennae all that it is necessary to do is to replace one-half of the Hertzian oscillator (Fig. 5) by a mesh held rigidly in a frame clamped to a table (Fig. 10) and to substitute a rod for the other sphere. Thus a simple model of a T antenna is readily built. By changing the position of the upper rod the antenna is transformed into the inverted L type, as illustrated in Fig. 11.

Mushroom form and other types of antennae can be reproduced in the same way by means of a few rods and connecting pieces available in every school laboratory. The vertical rod of the antenna is supported by a board fixed to the table by means of clamps. The transmitter is placed in a vertical position so as to enable one to observe the measur-

ing instruments. The following simple phenomena can be demonstrated with the models:

1. Maximum current in the antenna is obtained when the transmitter is exactly tuned to the antenna.

2. The wave-length of the antenna increases with the total length of the antenna, or with an increase in the number of rods at the top of the antenna.

3. The potential node of the antenna is situated at the point where the vertical wires are connected to the counterpoise frame. An insulated or earthed wire connected to this point does not affect the current indication of the antenna ammeter. The antenna current decreases if the wire is connected to any other point of the antenna system. The largest disturbance is produced by touching those parts of the antenna or of the counterpoise which are farthest away from the potential node.

26. *Models for Purposes of Investigation.*—For purposes of experimental research more elaborate constructions of antennae models are required. All those details of antennae systems met with in practical construction which may have any influence upon the electrical characteristics have to be considered in the model. Thus the height and the material of the supporting structure, neighboring dielectric or conductive masses, electric properties of the ground and other details may have to be taken into consideration.

27. *The Flat-Top Antenna.*—One of the models used for investigations is illustrated in Fig. 12. It is a model of a flat-top antenna supported on three masts. The base of the model consists of a wooden frame supported on four legs. The frame, covered underneath with a copper sheet, forms a tray which can be filled with any material of the proper conductivity to approximately represent the properties of the ground. Crosspieces can be shifted and clamped by means of wooden screws in any convenient position along the frame. Carriages are arranged so as to be moved along the crosspieces and to be likewise fixed in any convenient place. These carriages have vertically-bored threaded holes to receive the bases of the standards serving to support the antenna wires. The number of crosspieces and carriages to be used depends on the number of masts employed for supporting the antenna wires. In the figure two crosspieces and three carriages are shown with three masts. Hard rubber insulating pieces or rubber bands can be shifted along the masts to set the antenna wires at any desirable height above

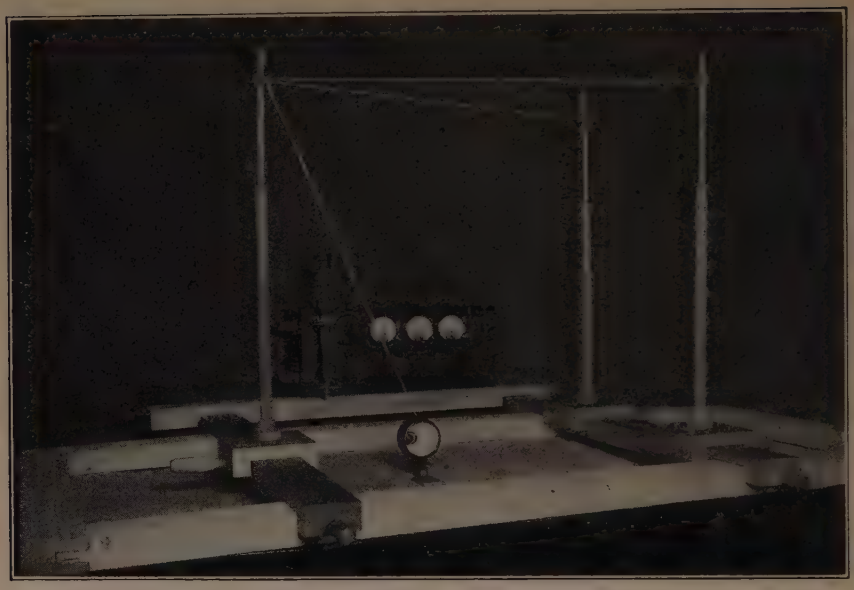

FIG. 12. MODEL OF A FLAT-TOP ANTENNA SUPPORTED ON THREE MASTS

the ground plate. Two antenna wires are visible in the figure ascending from the ammeter towards the top of one of the masts from which they diverge towards the top of the two other masts. The crosspieces have extensions, reaching 50 cm. beyond the frame, which are made to carry a baseboard for the short wave transmitter. By moving this board parallel to the frame the coupling of the transmitter with the antenna can be regulated. Finer adjustment of the coupling is carried out by means of the handle designed to rotate the plane of the transmitter loop. A similar model of a T antenna supported on two masts is shown in Fig. 13.

V. DETERMINATION OF ANTENNA CONSTANTS ON MODELS

28. *Choice of Scale and Antenna Type for the Verification of Similitude.*—It is possible to determine the chief characteristics of every type of antenna by means of a model of the kind described. The fundamental wave-length, the static as well as the dynamic capacity, the total and the radiation resistance can be determined, and from the values obtained by measurements, the inductance, the losses, and the efficiency of a given antenna can be calculated. However, the scale of the model

FIG. 13. MODEL OF A T ANTENNA

is limited on the one hand by the space available and on the other hand by the diameter of the wires that can be used for the antenna model. If the model is too large a special building is required to house the structure, if it is too small, wires of small diameter, possessing too small strength and introducing too large a resistance in the antenna circuit, become necessary.

For antennae corresponding in size to average ship antennae a model about 1/50 to 1/100 of the original may be used, but for the large antenna of a transatlantic station a scale of 1/250 to 1/500 may be practical.

It might appear advisable to begin the study of antenna constants on a model of a large type of antenna such as is used for purposes of transatlantic radio communication. These antennae are the most expensive, and should it be possible to pre-determine by means of models the principal data required for rational design of such extensive radiating systems, the economical advantage of this method would become apparent. There were, however, other considerations of an economic, technical, and scientific nature militating against this choice.

Transatlantic antennae types, being very expensive, will be built rarely. Their number is limited by the small size of our planet. If one considers the rapid progress made lately in the application of short waves for long distance radio communication, it appears doubtful whether a single one will be added to the number of existing transatlantic antennae.

It is necessary also to take into account the technical difficulties involved in making and handling such a model. Supposing a model of the type of antenna erected for the Long Island Transatlantic Radio Station had to be made. The length of the original antenna is about 3000 m., its height is 123 m. In order to keep the model of reasonable dimensions the latter would have to be made to a scale of 1/1000, with a length of 3 m. and a height of 0.123 m. This height is so small that instruments inserted would cause greater changes in the distribution of current along the wires than is admissible from the point of view of accuracy of measurement. The antenna wires would have to be of ex-extremely small diameter (0.012 mm.), and would therefore be very difficult to handle and possess excessive resistance. Other difficulties would arise in connection with the excitation of the model, which involves the production of short waves. To overcome these and other difficulties connected with models of transatlantic antennae the use of a larger scale would be necessary.

Moreover, a gradual development of the use of antenna models requires first of all an experimental verification of the principle of similitude for typical forms of antennae for which verification by theoretical investigation is not at the present time possible. For two limiting cases only can the correctness of the principle be demonstrated by theory—on the one hand for the straight wire oscillator with uniformly distributed capacity and inductance, on the other, for the closed circuit having lumped capacity and inductance. A review of the antenna types indicated in Fig. 1 shows that one of these limiting cases is approximately represented by the straight vertical Marconi antenna, while the large transatlantic antenna with its vertical part very small in comparison with the large horizontal part forming a large condenser with the earth's surface, approximately represents the other. Between these limits we find all the other types of antennae. It is clear that for the experimental verification of the principle of similitude it is important to choose an antenna type which is as far removed from these two limiting cases as possible.

This is the chief reason why in this investigation a T antenna was chosen as the type to be used in testing the correctness of the principle. If the principle is proved for this type of antenna with a degree of accuracy sufficient for engineering purposes, there will remain little doubt that this method is applicable for other antennae types.

Another reason for choosing the T antenna is that it represents most closely the type of transmitting antenna most commonly used for ships, land stations, and broadcasting stations.

The methods of measurement employed in connection with the determination of antennae constants are in principle identical with those used on full-sized antennae, but the technique had to be modified and adapted to the use of short waves and non-quasi-stationary circuits. Care had to be taken to avoid errors due to the leads connecting the loading condensers, coils, resistance elements, ammeters, etc., with the antenna, and to diminish to a minimum the influence of the body of the transmitter and of the observer upon the antenna.

29. *Auxiliary Apparatus.*—Some of the auxiliary apparatus used in the determination of antennae constants are shown in Fig. 14. The calibrated precision wavemeter *a* has been described already. For rough measurements, a simpler wavemeter construction *b*, consisting of a two-plate disc condenser and a loop of copper wire, was used. The standard loading condenser *c* was made up of a brass base-plate upon

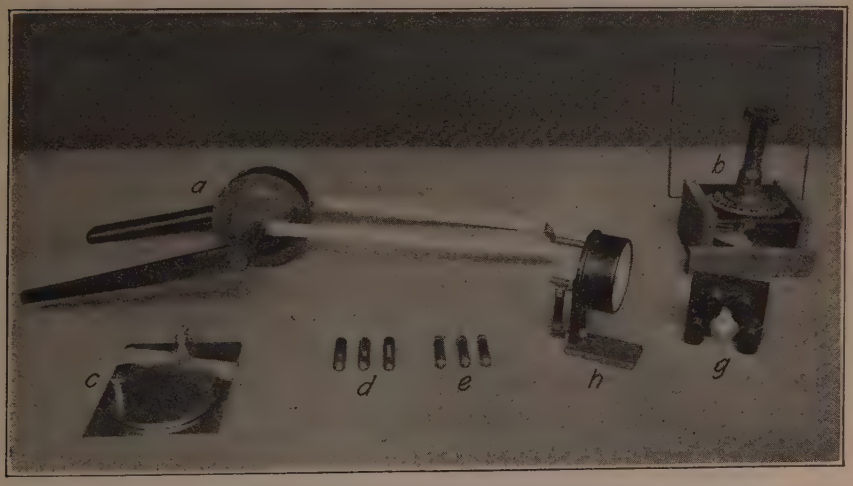

FIG. 14. AUXILIARY APPARATUS USED FOR THE DETERMINATION OF ANTENNA CONSTANTS

which a circular metallic disc 9 cm. in diameter was placed, supported on three amber pins the length of which can be adjusted from above. When the pins are adjusted to a length of 1 mm., this condenser has a capacity of 55 e. s. u. No leads whatever are required to insert the condenser at the base of the antenna model. The two binding posts shown in the figure are used only during the calibration with much longer wave-lengths. The loading coils *d* are wound on hard rubber rods 1 cm. in diameter, all of equal length and supplied with connecting knobs at the ends. The bodies for the resistance elements *e* are made of the same material and of the same dimensions. The resistance wire, however, is not wound on the rods but inserted in slots cut axially. Caps with holes in the center are screwed to the ends of the rods. Straight pieces of manganin wire, 1 to 4 cm. long and 0.05 mm. thick, with a resistivity of 2.33 ohm per cm. length are soldered to No. 24 copper wire endpieces, inserted through the cap holes into the slots. The copper wire ends are then soldered to the caps. The antenna current is measured by a thermo-couple *g* or by means of a Weston thermo-ammeter, combining a number of thermo-couples with a galvanometer in a compact instrument 7 cm. in diameter and 3.5 cm. wide. This instrument *h* was adapted to the measurement of antenna constants by supplying it with a brass base-plate insulated from the lower instrument terminal by a hard rubber support. The resistance elements *e* or the loading coils *d* can be inserted between the base-plate and lower ammeter terminal and are kept in position by means of contact springs.

30. *Measurement of Fundamental Wave-Length.*—In order to determine the natural wave-length of the antenna, an element of very low resistance, having no manganin wire but only copper wire, is inserted, the thermo-ammeter base-plate is placed directly on the antenna ground plate and the lower antenna end connected to the upper ammeter terminal. The condenser of the transmitter circuit is then adjusted for maximum current in the antenna model. The wavemeter (Fig. 9) is placed at a distance of about one-fourth wave-length from the antenna wire with the plane of its loop directed toward the vertical antenna wire. A slight decrease of the antenna current is noticed when the wavemeter is set in resonance with the emitted wave-length. Another and simpler method is to disconnect the antenna from the ground after it has been tuned and then measure the wave-length of the transmitter.

The accuracy of the antenna wave-length measurement depends largely on the damping of the particular circuit. Comparative measure-

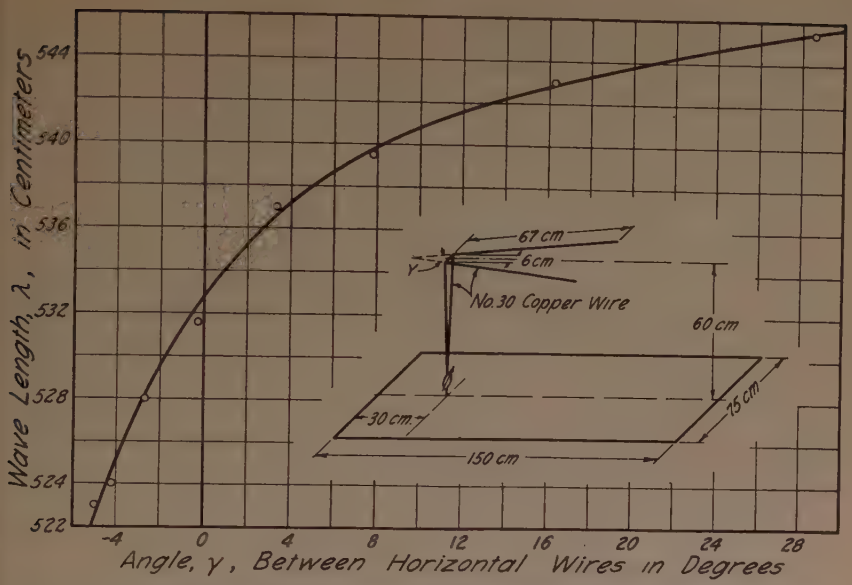

FIG. 15. CURVE SHOWING THE RELATION BETWEEN WAVE-LENGTH AND THE POSITION OF THE UPPER ANTENNA WIRES

ments on a one-hundredth scale model of a T antenna supported on wooden masts, as shown in Fig. 13, having its two horizontal wires 60 cm. above the ground plates, 75 cm. long, and separated 6 cm. from each other, gave 490.5 cm. for the natural wave-length while the original ship antenna represented by the model has a wave-length of 510 m. If, however, steel rods with guys were used to support the model antenna its wave-length increased to 509.5 cm. This value multiplied by the model coefficient 100 is in very close agreement with the wave-length of the full-sized antenna.

The precision with which wave-lengths of antennae can be determined by models suggests the use of the methods described for the study of the effect of the change of the relative position of the antenna wires on the wave-length and other constants. The curve of Fig. 15, giving the result of an investigation made on a model illustrated in Fig. 12, may serve as an example. The length of the wires and the height of the horizontal wires above the ground plate were kept constant, while the angle γ between the top wires was varied. New positions were obtained by separating the masts supporting the ends of the two horizontal wires,

then the natural wave-length of the antenna as determined by measuring that frequency of the oscillating transmitting circuit which produced maximum current in the antenna. The curve shows that the wave-length increases with the angle γ at a rate decreasing as this angle becomes larger. A variation in wave-length produced by a relative change of position of the wires of one degree can be detected.

31. *Measurement of Effective Capacity.*—The effective capacity of a model antenna is measured by determining the decrease of wave-length due to loading the antenna with a condenser of known capacity. Three conditions have to be observed in order to obviate serious errors: (a) the introduction of the condenser must not affect the geometry of the antenna; (b) it must be inserted at a point very near to the potential node; (c) its capacity must be much larger than the antenna capacity. By the use of a condenser such as described in Section 29 and shown in Fig. 13 these conditions are satisfied. The condenser, which is only 0.5 cm. high, is placed directly upon the antenna ground plate, the latter being in direct contact with the lower condenser plate. The base of the antenna ammeter (Fig. 14) is then placed upon the upper condenser plate. Thus a condenser with a capacity C about five times larger than that of the antenna is inserted in such a way as to leave the geometrical configuration and the distribution of the current in the antenna practically undisturbed, or at least insignificantly changed. The wave-length of the loaded antenna is then determined in exactly the same way as already described and the effective capacity, C_{eff}, calculated from the known relations

$$\frac{\lambda_1}{\lambda_2} = \frac{2\pi\sqrt{C_{eff}\,L}}{2\pi\sqrt{\frac{C_{eff}\,C}{C_{eff}+C}\,L}}$$

$$C_{eff} = C\left(\frac{\lambda_1{}^2}{\lambda_2{}^2} - 1\right)$$

In the case of the T antenna model, the following data were obtained:

$$C = 55 \text{ e. s. u.}$$
$$\lambda_1 = 509.5 \text{ cm.}$$
$$\lambda_2 = 469.7 \text{ cm.}$$

$$C_{eff} = 55\left(\frac{509.5^2}{469.7^2} - 1\right) = 9.68 \text{ e. s. u.}$$

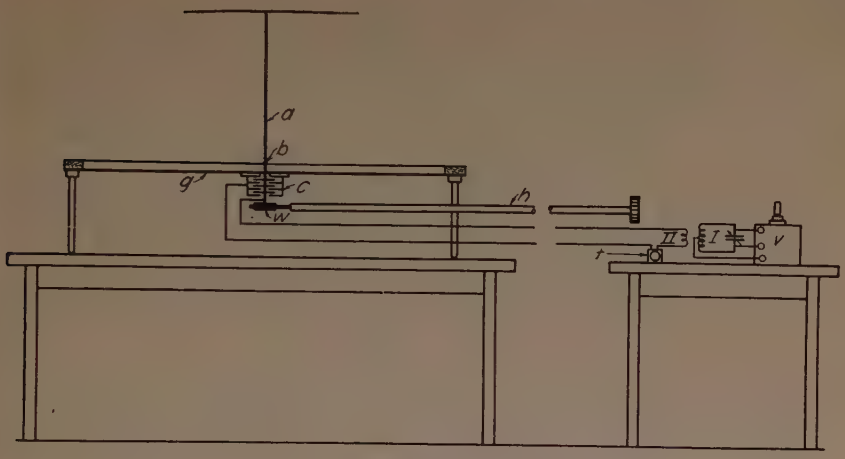

FIG. 16. DIAGRAM OF THE ARRANGEMENT FOR THE DETERMINATION OF STATIC CAPACITY OF ANTENNA MODELS

The effective capacity of an antenna is always smaller than its static capacity. It depends on the potential distribution along the antenna, increases with decreasing frequency used to excite the system, and approaches closely the value of the static capacity when the frequency of the loaded antenna becomes about ten times smaller than that of the unloaded.

32. *Measurement of Static Capacity.*—To determine the static capacity of the models, wave-lengths about 100 to 200 times longer than that of the unloaded antenna were used. The arrangement of the apparatus is shown in Fig. 16.

The procedure is as follows: A variable condenser C with the stationary discs attached directly beneath the antenna ground plate g is applied so that the ground plate forms a part of the capacity of the condenser. An extension of the movable disc system is passed through a hole in the ground plate, forming a binding post b for the antenna. By connecting this post with the antenna lead a the capacity of the condenser C is increased by an amount due to the added capacity of the antenna. This condenser forms a part of an oscillating secondary circuit II loosely coupled with a primary oscillating circuit I operated by thermionic vacuum tube v, at a constant wave-length of about 1000 m. The setting of the calibrated condenser C is adjusted by a long handle h attached to a

worm gear w. By means of a thermo-galvanometer t the resonance condenser setting is observed first without the antenna connection and then with the capacity of the antenna added to the circuit. The difference between these settings determines the capacity of the antenna, which is very nearly equal to its static value.

The static capacity of the T antenna model was measured by this method. Radio-frequency currents corresponding to a wave-length of 935 meters, that is, 186 times longer than the wave-length of the model, were used for the excitation of the antenna. The value obtained was

$$C_s = 11.95 \text{ e. s. u.}$$

for the antenna supported on wooden masts. If, however, steel rods 1.2 cm. in diameter placed directly on the ground plate and stayed by insulated guys of copper wire were used to support the antenna, the value

$$C_s = 14.6 \text{ e. s. u.}$$

was found. The static capacity of the full-sized ship antenna similar in form to the model, but 100 times larger, is known to be 1400 cm. The discrepancy of about +4.25 per cent may be due partly to the arrangement of the guys possibly differing from the actual conditions, and partly to the influence of the surrounding walls and objects in the laboratory where the measurements were made.

33. *Influence of Metallic Masses Placed in the Antenna Field.*— Metallic masses in the antenna field increase the antenna capacity and antenna wave-length. The methods of determination of antenna capacity were used to study the influence of metallic masts and guys. Table 2 gives the values of constants for wooden, steel-insulated, and steel-grounded masts, showing the increase of both the capacity and the wave-length due to the use of metallic supports, and also the increase of both due to the guys. As seen from the third column the ratio of the effective capacity to the static capacity is larger for metallic masts. Other observations on the models have shown an increase of these ratios for grounded steel masts as compared with steel masts insulated at the base. This indicates an improvement in the distribution of current along the antenna, because an approach of the ratio $\frac{C_{eff}}{C_s}$ to unity means that the effective height of the antenna approaches the value of its geometric height. This may offer an explanation of the ob-

TABLE 2

INFLUENCE OF MASTS AND GUYS UPON THE ANTENNA CONSTANTS

Mast	Effective Capacity C_{eff} e. s. u.	Static Capacity C_s e. s. u.	$\frac{C_{eff}}{C_s}$	Fundamental Wave-length λ_o cm.	Total Length of Antenna Wires l cm.	$\frac{\lambda_o}{l}$	Calculated Effective Inductance L_{eff} e. m. u.
Wooden	8.3	11.95	0.695	490.5	95	5.27	724
Steel Rods Insulated Without Guys		12.48			95		
Steel Rods Grounded Without Guys	9.68	13	0.745	509.5	95	5.36	670
Steel Rods Grounded With Insulated Copper Guys		14.6			95		

servation often made by radio engineers, that, in spite of losses due to induction of radio-frequency currents in the metallic supports of antennae, the range of the transmitting station does not suffer by it appreciably, even when the masts are grounded. The increase of effective height in these cases counterbalances the losses due to induction.

34. *Measurement of Total Effective Resistance.*—The total effective resistance of an antenna includes the resistance R_m causing the production of Joulian heat, dielectric as well as other losses, and the resistance S_m connected with the useful dissipation of energy in form of Hertzian waves.

The total resistance

$$R_t = R_m + S_m$$

is defined as the ratio E/I where E is the effective alternating electromotive force exciting the antenna, and I the effective current at the antinode of the antenna for the case of resonance.

This constant R_t can be measured on models by a method of resistance substitution identical in principle with that used on full-sized antennae. The current i_1 at the foot of the antenna model is observed, and then a resistance element (Fig. 14e) is introduced, increasing the total resistance by Δr and decreasing the initial current to i_2. Assuming constant electromotive forces, the currents are inversely proportional to the total resistances for each case

$$\frac{i_1}{i_2} = \frac{R_t + \Delta r}{R_t}$$

The total resistance is accordingly calculated from the formula

$$R_t = \frac{\Delta r}{\frac{i_1}{i_2} - 1}$$

35. *Current Distribution and Radiation Resistance.*—It is not so simple to measure that part of the total antenna resistance which is due to the radiation of Hertzian waves utilized in radio transmission. For wave-lengths larger than the geometric length of the antenna this constant S_m is given by the expression

$$S_m = 1579 \left(\frac{h_{eff}}{\lambda}\right)^2 \text{ ohms}$$

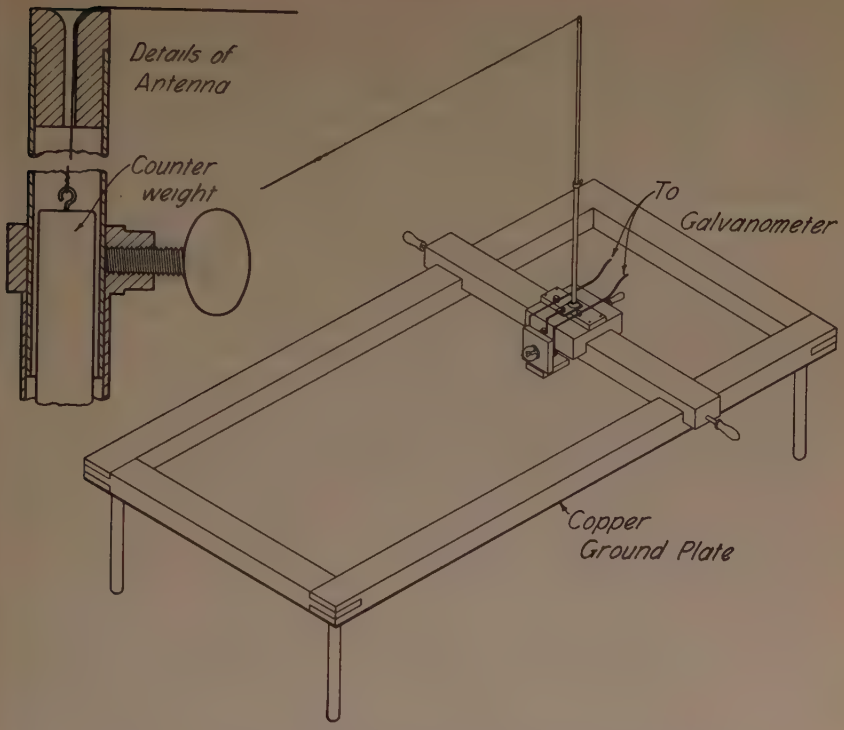

FIG. 17. RECEIVING ANTENNA FOR RADIATION MEASUREMENTS

The determination of the effective height h_{eff} presupposes the knowledge of current distribution along the antenna. Researches by the author aim at the development of a precise method for the experimental determination of current distribution. However, at the present time this method does not give sufficiently accurate results, and therefore other methods, based upon the effects produced by the radiation field of a transmitting antenna upon another receiving antenna placed at a certain safe distance from the transmitter, were used.

36. *Receiving Antenna for Measuring Radiation Resistance.*—A receiving antenna used in connection with transmitting antenna models is illustrated in Fig. 17. A wooden frame with a copper ground plate of exactly the same dimensions as used for models of transmitting antennae carries on a bakelite plate attached to a carriage a telescopic antenna consisting of two brass tubes sliding one into the other. The inner tube,

which may be fixed at any definite height by means of a set screw, carries at the top a brass cup with a funnel-like opening ending at the lower part in a fine hole. Through this hole a bronze wire slides out of the tube over the cup. The wire carries a counterweight moving inside the antenna tubes and can be pulled out by means of a cord attached to an elastic insulator fixed to the other end of the wire. The cord leads over a pulley to the place where observations are made and is used to tune the receiving antenna to the frequency of the transmitting antenna by altering the length of its horizontal part formed of the bronze wire. The base of the antenna tube is connected through the thermo-couple and resistance element (Fig. 14) to the ground plate. A pair of leads connects the thermo-couple with a sensitive galvanometer. Instead of an antenna a receiving loop was also used, consisting of a rectangular loop made of stiff wire supported on two standards. A condenser of the type described in Section 29 was inserted in the center of one of the vertical wires forming the loop, while a thermo-couple was inserted symmetrically in the other vertical wire. The handle of the condenser was turned by pulling cords leading to the observer.

37. *Measurement of Radiation Resistance by Varying the Height of the Antenna Model.*—The earliest method employed for radiation measurements was that suggested by Erskine Murray[12] for actual antennae which were coupled with quenched spark transmitters. This method was cumbersome, and beset with the difficulty of lowering the antennae without considerably disfiguring their geometrical shape. But it is comparatively easy to use this method of radiation measurements on models of those types of antennae for which the ohmic resistance is small in comparison with radiation resistance.

For a given receiving antenna, at a definite distance, the power absorbed by the receiving antenna is related to the power radiated by the transmitting antenna as shown by the equation,

$$S_m I^2 = A\, i_1^2$$

where A is a constant dependent on the total resistance of the receiving antenna and other transmission factors.

With the transmitting antenna lowered by a small amount, not exceeding 10 per cent of its geometric height, and its wave-length restored to the initial value, the radiation resistance S_m decreases by

an amount Δs, so that for the same value of antenna current I the power radiated, and also that received, will decrease proportionately. This involves a corresponding decrease of the current in the receiving antenna and consequently

$$(S_m - \Delta s)\, I^2 = A\, i_2^2$$

From these two equations the radiation resistance S_m can be calculated.

$$S_m = \frac{\Delta s}{I - \left(\frac{i_2}{i_1}\right)^2}$$

The value Δs can be determined by measuring the total antenna resistance R_t' before the antenna is changed in height and R_t'' after it is lowered.

$$\Delta s = R_t' - R_t''$$

The effective height of the antenna is then calculated from the relation

$$h_{eff} = \frac{\lambda \sqrt{S_m}}{39.7}$$

The measurements of radiation on such models must be carried out in a field free from buildings, trees, and overhead wires, within a radius of at least 10 wave-lengths in order to avoid reflection, refraction, and absorption of the waves. The first radiation measurements with models were undertaken on the grounds extending south of the Electrical Engineering Laboratory under very unfavorable conditions.

With the model of a T antenna as already described and shown in Fig. 13, but with the ground plate set directly on ground, the following results were obtained:

Radiation resistance measured.....	S_m =	14.8 ohms
Resistance of the thermo-ammeter measured.....	R_m =	4.6 "
Ohmic resistance of antenna bronze wires 5 mils (0.0125 cm.), calculated.....		3.3 "
Balance for the earth resistance.....		2.5 "
Total antenna resistance measured..	R_t =	25.2 "

Effective height of antenna

$$h_{eff} = \frac{\lambda \sqrt{S_m}}{39.7} = \frac{510 \sqrt{14.8}}{39.7} = 48.8 \text{ cm.}$$

Current distribution factor

$$\beta = \frac{h_{eff}}{h_g} = \frac{48.8}{60} = 0.81$$

The value of $S_m = 14.8$ ohms corresponds within 8 per cent to values of radiation resistance known for actual antennae of this type.

It must be taken into account that radiation measurements in general cannot be made with great accuracy by the present methods. Nor is the technique in connection with radiation measurements on models sufficiently developed at the present time to guarantee the same precision as achieved in capacity and wave-length measurements on models.

38. *Radiation Measurement by Means of Two Models Similar in Shape and Dimensions.*—A new method is being developed by the author which under better local conditions surrounding the models promises considerable improvement in accuracy.

This method is based on the use of perfectly similar models for the transmitting and the receiving antenna. If we apply the transmission formula, which is valid for short distances,

$$\frac{i}{I} = K \frac{h_1 \, h_2}{\lambda \, D \, R_t}$$

then, the antennae having similar geometrical dimensions

$$h_1 = h_2 = h \qquad h_1 \, h_2 = h^2$$

we obtain

$$h^2 = \frac{i}{I} \frac{\lambda \, D \, R_t}{120\pi}$$

The effective height is related to the radiation resistance S_m by the expression

$$h^2 = \frac{S_m \lambda^2}{160 \, \pi^2}$$

From the last two equations we obtain

$$S_m = \frac{i}{I} \frac{160\pi^2}{120\pi} \frac{DR_t}{\lambda} = 4.2 \frac{i}{I} \frac{DR_t}{\lambda}$$

Thus the radiation resistance S_m is determined by measuring the ratio of currents in the receiving and transmitting antenna, the total resistance R_t of the receiving antenna, the distance D, and the wave-length λ.

For the same purpose special instruments are being constructed permitting remote control of the tuning and measuring apparatus, in order to eliminate errors due to the proximity of the observer to the models.

39. *Independence of Model Operation of Radio Traffic.*—The frequency used in the operation of the antennae models is 50 to 3000 times higher than that applied for broadcasting and for other forms of radio traffic. The resulting difference of wave-length is large enough to exclude any interference whatsoever with other radio stations working at the same time. Even in cases where the antennae models were operated in the same building with the broadcasting station of the University of Illinois, no mutual influence could be observed. It is possible therefore to carry out the investigation and to demonstrate the antennae models at any time independently of the operation of even the nearest stations.

40. *Summary and Conclusions.*—From the investigations carried out with model antennae the following conclusions may be drawn:

(a) The characteristics of full-sized antennae can be predicted by means of miniature models to a scale of about one to one hundred, excited by proportionately short waves (from three to six meters long).

(b) The wave-length, static capacity, effective capacity, and effective height of the model can be determined with sufficient accuracy for the design of full-sized antenna installations.

(c) The relation of these characteristic constants of full-sized antennae to the corresponding values as determined with accurate models is the simple ratio of linear dimensions, noted as m in this discussion.

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(d) The determinations of radiation resistance by means of models approximate measured values for actual antennae. The slight discrepancies are believed to be due chiefly to the presence of interfering objects.

(e) Special simple forms of adjustable model antennae permit the rapid comparison of the effects of variations of form or dimensions on the performance of full-sized antennae. This type of model serves for the purpose of demonstrating the properties of various types of antennae.

(f) The results of any suggested alteration in the form or dimension of full-sized antennae are convincingly shown by means of model antennae leading rapidly and economically to the production of the most satisfactory structure of any general type.

(g) A new type of condenser and a wave-meter have been developed permitting remarkable precision of measurement of the two- to ten-meter waves.

(h) A wavemeter bridge permits the visual demonstration of actual nodes and loops of short waves illustrating through mechanical analogy the existence of standing waves and phase relations and the effect of varying the characteristic factors in oscillation circuits.

(i) The use of these exceedingly short waves (less than ten meters) cannot disturb commercial radio traffic and obviates possible errors from interference of radio stations.

(j) The distribution of potential along antennae wires of models, along Hertzian oscillators, and along closed circuits can be studied by observing the variation of resonance current due to small discs or short wires shifted along the conductor investigated.

APPENDIX A

Capacity of a Stepped Disc Condenser

In order to calculate the capacity of the condenser described in Section 21 let us consider the areas and the respective distances separating the surfaces of two overlapping stepped discs. In Fig. 18a let the diametrical edge of the step for the stationary disc be represented by the line AB, the edge for the rotating disc in any arbitrary position by the line CD, making an angle α with AB. These lines divide the area of the discs into four segments a_1, a_2, a_3, and a_4. The cross-sections through a_1-a_2 and a_3-a_4 are shown in Fig. 18b. If d is the diameter of the disc, the respective areas will be

$$a_1 = a_2 = \frac{d^2\pi}{4}\,\frac{\alpha}{360} \quad \text{and} \quad a_3 = a_4 \quad \frac{d^2\pi}{4}\,\frac{180-\alpha}{360}$$

and the distances separating the upper from the lower planes will be

$$\delta_1 = \delta \quad \delta_2 = n\delta \quad \delta_3 = \delta_4 = \frac{n\delta}{2} + \frac{\delta}{2} = \frac{\delta}{2}(n+1)$$

where δ and $n\delta$ are the respective distances between the nearest and furthest planes.

The capacity of the condenser is given by the sum of the capacities of its sections, consequently

$$C = \frac{d^2\pi}{4}\,\frac{\alpha}{360}\,\frac{1}{4\pi\delta} + \frac{d^2\pi}{4}\,\frac{\alpha}{360}\,\frac{1}{4\pi n\delta} + 2\,\frac{d^2\pi}{4}\,\frac{180-\alpha}{360}\,\frac{1}{4\pi\frac{\delta}{2}(n+1)}$$

or

$$C = \frac{1}{8}\,\frac{d^2}{\delta(n+1)} + \frac{\alpha}{16}\,\frac{d^2}{360\,\delta n}\,\frac{(n-1)^2}{n+1}$$

This expression, which gives the total capacity of the pair of discs in electrostatic units, consists of two terms. One is independent of the relative position of the discs and the other is a linear function of α.

The limits within which the capacity of this type of condenser can be varied by angular displacement of the rotating part are obtained by

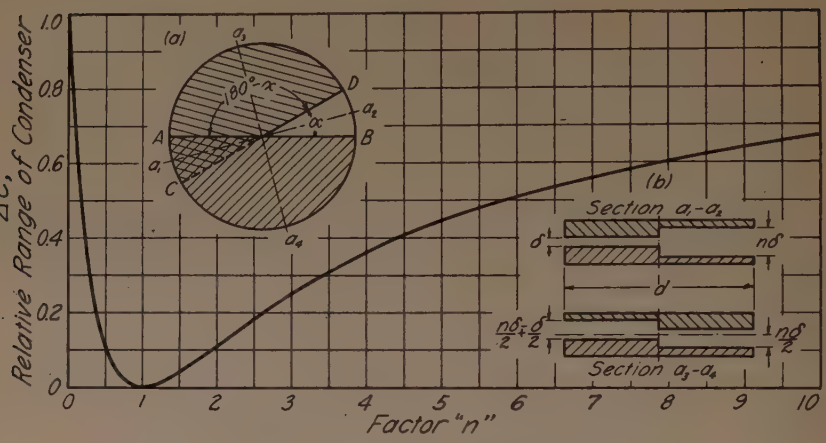

FIG. 18. CAPACITY RANGE FOR STEPPED DISC CONDENSER

evaluating the above expression for $a = 0$ deg. and for $a = 180$ deg. respectively. This gives for the lower limit

$$C_{\min} = \frac{1}{8} \frac{d^2}{\delta (n+1)}$$

and for the upper limit

$$C_{\max} = \frac{1}{8} \frac{d^2}{\delta (n+1)} + \frac{1}{16} \frac{180}{360} \frac{d^2}{\delta} \frac{(n-1)^2}{n+1} = \frac{1}{32} \frac{d^2}{n} \frac{(n-1)^2}{n+1}$$

The ratio of the maximum and minimum values for the capacity of the condenser is thus

$$\frac{C_{\max}}{C_{\min}} = \frac{\frac{1}{32} \frac{d^2}{\delta} \frac{(n-1)^2}{n+1}}{\frac{1}{8} \frac{d^2}{(n+1)}} = \frac{(n+1)^2}{4n}$$

depending exclusively upon the factor n.

The chief characteristic of the condenser becomes evident by investigating the expression for its range. The latter is given by the difference $C_{\max} - C_{\min}$, so that

$$\Delta C = C_{\max} - C_{\min} = C_{\max}\left(1 - \frac{C_{\min}}{C_{\max}}\right) = C_{\max}\left(1 - \frac{4n}{(n+1)^2}\right)$$

The graphical representation of this characteristic is given in Fig. 18 for $C_{max} = 1$. It consists of two divergent branches with their origin at the point $n = 1, \Delta C = 0$, both branches rising towards the limiting value of $\Delta C = 1$. Physically $n = 1$ means that the distances between the plates over both areas of the disc are made equal in magnitude and therefore, the range ΔC being zero in this case, no variation whatever can be attained by rotating the movable disc. With increasing n, however, the range increases and reaches its maximum value $\Delta C = 1$ for $n = \infty$. This case corresponds to a condenser having one-half of each disc removed altogether. The disc becomes semicircular and the condenser acquires the usual form. The standard form represents thus a special case of the new condenser type when $n = \infty$. According to the left branch of the characteristic for values of n decreasing from $n = 1$ to $n = 0$, the range increases also towards $\Delta C = 1$. This serves to verify the fact that it does not matter which of the two planes of the rotating stepped disc is further away from the stationary disc. If the ratio of the plate distances $\frac{\delta_1}{\delta_2}$ is taken reciprocally, so that $n = \frac{\delta_2}{\delta_1}$ the value ΔC remains unchanged because in the expression for ΔC

$$\frac{4n}{(n+1)^2} = \frac{4\frac{1}{n}}{\left(\frac{1}{n}+1\right)^2}$$

Two examples may serve to illustrate the accuracy with which the variation of capacity may be effected by means of this stepped-plate condenser. For a condenser having a maximum capacity of 16.2 e. s. u. the variation of capacity per degree of angular displacement will be

$$\Delta c = \frac{C}{180} = \frac{16.2}{180}\left[1 - \frac{4n}{(n+1)^2}\right]$$

If for n the value 10 is chosen this variation is

$$\Delta c = \frac{16.2}{180}\left[1 - \frac{40}{121}\right] = 0.06 \text{ e. s. u./degree}$$

For a condenser of the same maximum capacity, but with plate distances corresponding to $n = 2$

$$\Delta c = \frac{16.2}{180}\left[1 - \frac{8}{9}\right] = 0.01 \text{ e. s. u./degree}$$

It is thus possible to produce very small variations of capacity of the condenser by comparatively large angular displacements of the adjusting handle. The same principle is applicable to cylindrical condensers which are now in the course of construction.

APPENDIX B

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